

QUIVER GAUGE THEORY AND UNIFICATION AT ABOUT 4 TEV

P.H. FRAMPTON

*Department of Physics and Astronomy,
University of North Carolina at Chapel Hill,
Chapel Hill, NC 27599-3255, USA.
E-mail: frampton@physics.unc.edu*

The use of the AdS/CFT correspondence to arrive at quiver gauge field theories is discussed, focusing on the orbifolded case without supersymmetry. An abelian orbifold with the finite group Z_p can give rise to a $G = SU(N)^p$ gauge group with chiral fermions and complex scalars in different bi-fundamental representations of G . The precision measurements at the Z resonance suggest the values $p = 12$ and $N = 3$, and a unifications scale $M_U \sim 4$ TeV. The robustness and predictivity of such grand unification is discussed.

1. Quiver Gauge Theory

The relationship of the Type IIB superstring to conformal gauge theory in $d = 4$ gives rise to an interesting class of gauge theories. Choosing the simplest compactification¹ on $AdS_5 \times S_5$ gives rise to an $N = 4$ $SU(N)$ gauge theory which is known to be conformal due to the extended global supersymmetry and non-renormalization theorems. All of the RGE β -functions for this $N = 4$ case are vanishing in perturbation theory. It is possible to break the $N = 4$ to $N = 2, 1, 0$ by replacing S_5 by an orbifold S_5/Γ where Γ is a discrete group with $\Gamma \subset SU(2), \subset SU(3), \not\subset SU(3)$ respectively.

In building a conformal gauge theory model^{2,3,4}, the steps are: (1) Choose the discrete group Γ ; (2) Embed $\Gamma \subset SU(4)$; (3) Choose the N of $SU(N)$; and (4) Embed the Standard Model $SU(3) \times SU(2) \times U(1)$ in the resultant gauge group $\otimes SU(N)^p$ (quiver node identification). Here we shall look only at abelian $\Gamma = Z_p$ and define $\alpha = \exp(2\pi i/p)$. It is expected from the string-field duality that the resultant field theory is conformal in the $N \rightarrow \infty$ limit, and will have a fixed manifold, or at least a fixed point, for N finite.

Before focusing on $N = 0$ non-supersymmetric cases, let us first examine an $N = 1$ model first put forward in the work of Kachru and Silverstein⁵.

The choice is $\Gamma = Z_3$ and the $\mathbf{4}$ of $SU(4)$ is $\mathbf{4} = (1, \alpha, \alpha, \alpha^2)$. Choosing $N=3$ this leads to the three chiral families under $SU(3)^3$ trinification⁶

$$(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3) \quad (1)$$

In this model it is interesting that the number of families arises as $4-1=3$, the difference between the 4 of $SU(4)$ and $N = 1$, the number of unbroken supersymmetries. However this model has no gauge coupling unification; also, keeping $N = 1$ supersymmetry is against the spirit of the conformality approach. We now present an example which accommodates three chiral families, break all supersymmetries ($N = 0$) and possess gauge coupling unification, including the correct value of the electroweak mixing angle.

Choose $\Gamma = Z_7$, embed the 4 of $SU(4)$ as $(\alpha^2, \alpha^2, \alpha^{-3}, \alpha^{-1})$, and choose $N=3$ to aim at a trinification $SU(3)_C \times SU(3)_W \times SU(3)_H$.

The seven nodes of the quiver diagram will be identified as C-H-W-H-H-H-W.

The behavior of the 4 of $SU(4)$ implies that the bifundamentals of chiral fermions are in the representations

$$\sum_{j=1}^7 [2(N_j, \bar{N}_{j+2}) + (N_j, \bar{N}_{j-3}) + (N_j, \bar{N}_{j-1})] \quad (2)$$

Embedding the C, W and H $SU(3)$ gauge groups as indicated by the quiver mode identifications then gives the seven quartets of irreducible representations

$$\begin{aligned} & [3(3, \bar{3}, 1) + (3, 1, \bar{3})]_1 + \\ & + [3(1, 1, 1 + 8) + (\bar{3}, 1, 3)]_2 + \\ & + [3(1, 3, \bar{3}) + (1, 1 + 8, 1)]_3 + \\ & + [(2(1, 1, 1 + 8) + (1, \bar{3}, 3) + (\bar{3}, 1, 3)]_4 + \\ & + [2(1, 1, 1 + 8) + 2(1, \bar{3}, 3)]_5 + \\ & + [2(\bar{3}, 1, 3) + (1, 1, 1 + 8) + (1, \bar{3}, 3)]_6 + \\ & + [4(1, 3, \bar{3})]_7 \end{aligned} \quad (3)$$

Combining terms gives, aside from (real) adjoints and overall singlets

$$3(3, \bar{3}, 1) + 4(\bar{3}, 1, 3) + (3, 1, \bar{3}) + 7(1, 3, \bar{3}) + 4(1, \bar{3}, 3) \quad (4)$$

Cancelling the real parts (which acquire Dirac masses at the conformal symmetry breaking scale) leaves under trinification $SU(3)_C \times SU(3)_W \times SU(3)_H$

$$3[(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)] \quad (5)$$

which are the desired three chiral families.

Given the embedding of Γ in $SU(4)$ it follows that the 6 of $SU(4)$ transforms as $(\alpha^4, \alpha, \alpha, \alpha^{-1}, \alpha^{-1}, \alpha^{-4})$. The complex scalars therefore transform as

$$\sum_{j=1}^7 [(N_j, \bar{N}_{j\pm 4}) + 2(N_j, \bar{N}_{j\pm 1})] \quad (6)$$

These bifundamentals can by their VEVs break the symmetry $SU(3)^7 = SU(3)_C \times SU(3)_W^2 \times SU(3)_H^4$ down to the appropriate diagonal subgroup $SU(3)_C \times SU(3)_W \times SU(3)_H$.

Now to the final aspect of this model which is its motivation, the gauge coupling unification. The embedding in $SU(3)^7$ of $SU(3)_C \times SU(3)_W^2 \times SU(3)_H^4$ means that the couplings $\alpha_1, \alpha_2, \alpha_3$ are in the ratio $\alpha_1/\alpha_2/\alpha_3 = 1/2/4$. Using the phenomenological data given at the beginning, this implies that $\sin^2\theta = 0.231$. On the other hand, the QCD coupling is $\alpha_3 = 0.0676$ which is too low unless the conformal scale is at least 10TeV.

2. Gauge Couplings.

An alternative to conformality, grand unification with supersymmetry, leads to an impressively accurate gauge coupling unification⁷. In particular it predicts an electroweak mixing angle at the Z-pole, $\sin^2\theta = 0.231$. This result may, however, be fortuitous, but rather than abandon gauge coupling unification, we can rederive $\sin^2\theta = 0.231$ in a different way by embedding the electroweak $SU(2) \times U(1)$ in $SU(N) \times SU(N) \times SU(N)$ to find $\sin^2\theta = 3/13 \simeq 0.231^{4,8}$. This will be a common feature of the models in this paper.

The conformal theories will be finite without quadratic or logarithmic divergences. This requires appropriate equal number of fermions and bosons which can cancel in loops and which occur without the necessity of space-time supersymmetry. As we shall see in one example, it is possible to combine spacetime supersymmetry with conformality but the latter is the driving principle and the former is merely an option: additional fermions and scalars are predicted by conformality in the TeV range^{4,8}, but in general these particles are different and distinguishable from supersymmetric partners. The boson-fermion cancellation is essential for the cancellation of infinities, and will play a central role in the calculation of the cosmological constant (not discussed here). In the field picture, the cosmological constant measures the vacuum energy density.

What is needed first for the conformal approach is a simple model.

Here we shall focus on abelian orbifolds characterised by the discrete group Z_p . Non-abelian orbifolds will be systematically analysed elsewhere.

The steps in building a model for the abelian case (parallel steps hold for non-abelian orbifolds) are:

- (1) Choose the discrete group Γ . Here we are considering only $\Gamma = Z_p$. We define $\alpha = \exp(2\pi i/p)$.
- (2) Choose the embedding of $\Gamma \subset SU(4)$ by assigning $\mathbf{4} = (\alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ such that $\sum_{q=1}^{q=4} A_q = 0 \pmod{p}$. To break $N = 4$ supersymmetry to $N = 0$ (or $N = 1$) requires that none (or one) of the A_q is equal to zero \pmod{p} .
- (3) For chiral fermions one requires that $\mathbf{4} \not\equiv \mathbf{4}^*$ for the embedding of Γ in $SU(4)$.

The chiral fermions are in the bifundamental representations of $SU(N)^p$

$$\sum_{i=1}^{i=p} \sum_{q=1}^{q=4} (N_i, \bar{N}_{i+A_q}) \quad (7)$$

If $A_q = 0$ we interpret (N_i, \bar{N}_i) as a singlet plus an adjoint of $SU(N)_i$.

- (4) The $\mathbf{6}$ of $SU(4)$ is real $\mathbf{6} = (a_1, a_2, a_3, -a_1, -a_2, -a_3)$ with $a_1 = A_1 + A_2$, $a_2 = A_2 + A_3$, $a_3 = A_3 + A_1$ (recall that all components are defined modulo p). The complex scalars are in the bifundamentals

$$\sum_{i=1}^{i=p} \sum_{j=1}^{j=3} (N_i, \bar{N}_{i \pm a_j}) \quad (8)$$

The condition in terms of a_j for $N = 0$ is $\sum_{j=1}^{j=3} (\pm a_j) \neq 0 \pmod{p}^2$.

- (5) Choose the N of $\bigotimes_i SU(N d_i)$ (where the d_i are the dimensions of the representations of Γ). For the abelian case where $d_i \equiv 1$, it is natural to choose $N = 3$ the largest $SU(N)$ of the standard model (SM) gauge group. For a non-abelian Γ with $d_i \neq 1$ the choice $N = 2$ would be indicated.
- (6) The p quiver nodes are identified as color (C), weak isospin (W) or a third $SU(3)$ (H). This specifies the embedding of the gauge group $SU(3)_C \times SU(3)_W \times SU(3)_H \subset \bigotimes SU(N)^p$.

This quiver node identification is guided by (7), (8) and (9) below.

- (7) The quiver node identification is required to give three chiral families under Eq.(7) It is sufficient to make three of the $(C + A_q)$ to be W and the fourth H, given that there is only one C quiver node, so that there are three $(3, \bar{3}, 1)$. Provided that $(\bar{3}, 3, 1)$ is avoided by the $(C - A_q)$ being H, the remainder of the three family

trification will be automatic by chiral anomaly cancellation. Actually, a sufficient condition for three families has been given; it is necessary only that the difference between the number of $(3 + A_q)$ nodes and the number of $(3 - A_q)$ nodes which are W is equal to three.

- (8) The complex scalars of Eq. (8) must be sufficient for their vacuum expectation values (VEVs) to spontaneously break $SU(3)^p \longrightarrow SU(3)_C \times SU(3)_W \times SU(3)_H \longrightarrow SU(3)_C \times SU(2)_W \times U(1)_Y \longrightarrow SU(3)_C \times U(1)_Q$.

Note that, unlike grand unified theories (GUTs) with or without supersymmetry, the Higgs scalars are here prescribed by the conformality condition. This is more satisfactory because it implies that the Higgs sector cannot be chosen arbitrarily, but it does make model building more interesting

- (9) Gauge coupling unification should apply at least to the electroweak mixing angle $\sin^2\theta = g_Y^2/(g_2^2 + g_Y^2) \simeq 0.231$. For trification $Y = 3^{-1/2}(-\lambda_{8W} + 2\lambda_{8H})$ so that $(3/5)^{1/2}Y$ is correctly normalized. If we make $g_Y^2 = (3/5)g_1^2$ and $g_2^2 = 2g_1^2$ then $\sin^2\theta = 3/13 \simeq 0.231$ with sufficient accuracy.

We now answer all these steps for the choice $\Gamma = Z_p$ for successive $p = 2, 3, \dots$ up to $p = 7$.

- **p = 2**

In this case $\alpha = -1$ and therefore one cannot construct any complex $\mathbf{4}$ of $SU(4)$ with $\mathbf{4} \not\equiv \mathbf{4}^*$. Chiral fermions are therefore impossible.

- **p = 3**

The only possibilities are $A_q = (1, 1, 1, 0)$ or $A_q = (1, 1, -1, -1)$. The latter is real and leads to no chiral fermions. The former leaves $N = 1$ supersymmetry and is a simple three-family model⁵ by the quiver node identification C - W - H. The scalars $a_j = (1, 1, 1)$ are sufficient to spontaneously break to the SM. Gauge coupling unification is, however, missing since $\sin^2\theta = 3/8$, in bad disagreement with experiment.

- **p = 4**

The only complex $N = 0$ choice is $A_q = (1, 1, 1, 1)$. But then $a_j = (2, 2, 2)$ and any quiver node identification such as C - W

- H - H has 4 families and the scalars are insufficient to break spontaneously the symmetry to the SM gauge group.

- **p = 5**

The two inequivalent complex choices are $A_q = (1, 1, 1, 2)$ and $A_q = (1, 3, 3, 3)$. By drawing the quiver, however, and using the rules for three chiral families given in (7) above, one finds that the node identification and the prescription of the scalars as $a_j = (2, 2, 2)$ and $a_j = (1, 1, 1)$ respectively does not permit spontaneous breaking to the standard model.

- **p = 6**

Here we can discuss three inequivalent complex possibilities as follows:

(6A) $A_q = (1, 1, 1, 3)$ which implies $a_j = (2, 2, 2)$.

Requiring three families means a node identification C - W - X - H - X - H where X is either W or H. But whatever we choose for the X the scalar representations are insufficient to break $SU(3)^6$ in the desired fashion down to the SM. This illustrates the difficulty of model building when the scalars are not in arbitrary representations.

(6B) $A_q = (1, 1, 2, 2)$ which implies $a_j = (2, 3, 3)$.

Here the family number can be only zero, two or four as can be seen by inspection of the A_q and the related quiver diagram. So (6B) is of no phenomenological interest.

(6C) $A_q = (1, 3, 4, 4)$ which implies $a_j = (1, 1, 4)$.

Requiring three families needs a quiver node identification which is of the form *either* C - W - H - H - W - H *or* C - H - H - W - W - H. The scalar representations implied by $a_j = (1, 1, 4)$ are, however, easily seen to be insufficient to do the required spontaneous symmetry breaking (S.S.B.) for both of these identifications.

- **p = 7**

Having been stymied mainly by the rigidity of the scalar representation for all $p \leq 6$, for $p = 7$ there are the first cases which work. Six inequivalent complex embeddings of $Z_7 \subset SU(4)$ require consideration.

(7A) $A_q = (1, 1, 1, 4) \implies a_j = (2, 2, 2)$

For the required nodes C - W - X - H - H - X - H the scalars are *insufficient* for S.S.B.

$$(7B) \ A_q = (1, 1, 2, 3) \implies a_j = (2, 3, 3)$$

The node identification C - W - H - W - H - H - H leads to a *successful* model.

$$(7C) \ A_q = (1, 2, 2, 2) \implies a_j = (3, 3, 3)$$

Choosing C - H - W - X - X - H - H to derive three families, the scalars *fail* in S.S.B.

$$(7D) \ A_q = (1, 3, 5, 5) \implies a_j = (1, 1, 3)$$

The node choice C - W - H - H - H - W - H leads to a *successful* model. This is Model A of ⁸.

$$(7E) \ A_q = (1, 4, 4, 5) \implies a_j = (1, 2, 2)$$

The nodes C - H - H - H - W - W - H are *successful*.

$$(7F) \ A_q = (2, 4, 4, 4) \implies a_j = (1, 1, 1)$$

Scalars *insufficient* for S.S.B.

The three successful models (7B), (7D) and (7E) lead to an $\alpha_3(M) \simeq 0.07$. Since $\alpha_3(1\text{TeV}) \geq 0.10$ it suggests a conformal scale $M \simeq 10 \text{ TeV}$ ⁸. The above models have less generators than an $E(6)$ GUT and thus $SU(3)$ ⁷ merits further study. It is possible, and under investigation, that non-abelian orbifolds will lead to a simpler model.

For such field theories it is important to establish the existence of a fixed manifold with respect to the renormalization group. It could be a fixed line but more likely, in the $N = 0$ case, a fixed point. It is known that in the $N \longrightarrow \infty$ limit the theories become conformal, but although this 't Hooft limit is where the field-string duality is derived we know that finiteness survives to finite N in the $N = 4$ case⁹ and this makes it plausible that at least a conformal point occurs also for the $N = 0$ theories with $N = 3$ derived above.

The conformal structure cannot by itself predict all the dimensionless ratios of the standard model such as mass ratios and mixing angles because these receive contributions, in general, from soft breaking of conformality. With a specific assumption about the pattern of conformal symmetry breaking, however, more work should lead to definite predictions for such quantities.

3. 4 TeV Grand Unification

Conformal invariance in two dimensions has had great success in comparison to several condensed matter systems. It is an interesting question whether conformal symmetry can have comparable success in a four-dimensional description of high-energy physics.

Even before the standard model (SM) $SU(2) \times U(1)$ electroweak theory was firmly established by experimental data, proposals were made^{10,11} of models which would subsume it into a grand unified theory (GUT) including also the dynamics¹² of QCD. Although the prediction of $SU(5)$ in its minimal form for the proton lifetime has long ago been excluded, *ad hoc* variants thereof¹³ remain viable. Low-energy supersymmetry improves the accuracy of unification of the three 321 couplings^{14,7} and such theories encompass a “desert” between the weak scale ~ 250 GeV and the much-higher GUT scale $\sim 2 \times 10^{16}$ GeV, although minimal supersymmetric $SU(5)$ is by now ruled out¹⁵.

Recent developments in string theory are suggestive of a different strategy for unification of electroweak theory with QCD. Both the desert and low-energy supersymmetry are abandoned. Instead, the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group is embedded in a semi-simple gauge group such as $SU(3)^N$ as suggested by gauge theories arising from compactification of the IIB superstring on an orbifold $AdS_5 \times S^5/\Gamma$ where Γ is the abelian finite group Z_N^2 . In such nonsupersymmetric quiver gauge theories the unification of couplings happens not by logarithmic evolution¹² over an enormous desert covering, say, a dozen orders of magnitude in energy scale. Instead the unification occurs abruptly at $\mu = M$ through the diagonal embeddings of 321 in $SU(3)^N$. The key prediction of such unification shifts from proton decay to additional particle content, in the present model at $\simeq 4$ TeV.

Let me consider first the electroweak group which in the standard model is still un-unified as $SU(2) \times U(1)$. In the 331-model^{16,17} where this is extended to $SU(3) \times U(1)$ there appears a Landau pole at $M \simeq 4$ TeV because that is the scale at which $\sin^2\theta(\mu)$ slides to the value $\sin^2(M) = 1/4$. It is also the scale at which the custodial gauged $SU(3)$ is broken in the framework of¹⁸.

Such theories involve only electroweak unification so to include QCD I examine the running of all three of the SM couplings with μ as explicated in *e.g.*⁷. Taking the values at the Z-pole $\alpha_Y(M_Z) = 0.0101$, $\alpha_2(M_Z) = 0.0338$, $\alpha_3(M_Z) = 0.118 \pm 0.003$ (the errors in $\alpha_Y(M_Z)$ and $\alpha_2(M_Z)$ are less than 1%) they are taken to run between M_Z and M according to the SM

equations

$$\begin{aligned}\alpha_Y^{-1}(M) &= (0.01014)^{-1} - (41/12\pi)\ln(M/M_Z) \\ &= 98.619 - 1.0876y\end{aligned}\tag{9}$$

$$\begin{aligned}\alpha_2^{-1}(M) &= (0.0338)^{-1} + (19/12\pi)\ln(M/M_Z) \\ &= 29.586 + 0.504y\end{aligned}\tag{10}$$

$$\begin{aligned}\alpha_3^{-1}(M) &= (0.118)^{-1} + (7/2\pi)\ln(M/M_Z) \\ &= 8.474 + 1.114y\end{aligned}\tag{11}$$

where $y = \log(M/M_Z)$.

The scale at which $\sin^2\theta(M) = \alpha_Y(M)/(\alpha_2(M) + \alpha_Y(M))$ satisfies $\sin^2\theta(M) = 1/4$ is found from Eqs.(9,10) to be $M \simeq 4$ TeV as stated in the introduction above.

I now focus on the ratio $R(M) \equiv \alpha_3(M)/\alpha_2(M)$ using Eqs.(10,11). I find that $R(M_Z) \simeq 3.5$ while $R(M_3) = 3$, $R(M_{5/2}) = 5/2$ and $R(M_2) = 2$ correspond to $M_3, M_{5/2}, M_2 \simeq 400\text{GeV}, 4\text{TeV}, \text{and } 140\text{TeV}$ respectively. The proximity of $M_{5/2}$ and M , accurate to a few percent, suggests strong-electroweak unification at $\simeq 4$ TeV.

There remains the question of embedding such unification in an $SU(3)^N$ of the type described in ^{2,8}. Since the required embedding of $SU(2)_L \times U(1)_Y$ into an $SU(3)$ necessitates $3\alpha_Y = \alpha_H$ the ratios of couplings at $\simeq 4$ TeV is: $\alpha_{3C} : \alpha_{3W} : \alpha_{3H} :: 5 : 2 : 2$ and it is natural to examine $N = 12$ with diagonal embeddings of Color (C), Weak (W) and Hypercharge (H) in $SU(3)^2, SU(3)^5, SU(3)^5$ respectively.

To accomplish this I specify the embedding of $\Gamma = Z_{12}$ in the global $SU(4)$ R-parity of the $N = 4$ supersymmetry of the underlying theory. Defining $\alpha = \exp(2\pi i/12)$ this specification can be made by $\mathbf{4} \equiv (\alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ with $\Sigma A_\mu = 0 \pmod{12}$ and all $A_\mu \neq 0$ so that all four supersymmetries are broken from $N = 4$ to $N = 0$.

Having specified A_μ I calculate the content of complex scalars by investigating in $SU(4)$ the $\mathbf{6} \equiv (\alpha^{a_1}, \alpha^{a_2}, \alpha^{a_3}, \alpha^{-a_3}, \alpha^{-a_2}, \alpha^{-a_1})$ with $a_1 = A_1 + A_2, a_2 = A_2 + A_3, a_3 = A_3 + A_1$ where all quantities are defined (mod 12).

Finally I identify the nodes (as C, W or H) on the dodecahedral quiver such that the complex scalars

$$\Sigma_{i=1}^{i=3} \Sigma_{\alpha=1}^{\alpha=12} (N_\alpha, \bar{N}_{\alpha \pm a_i})\tag{12}$$

are adequate to allow the required symmetry breaking to the $SU(3)^3$ diagonal subgroup, and the chiral fermions

$$\Sigma_{\mu=1}^{\mu=4} \Sigma_{\alpha=1}^{\alpha=12} (N_{\alpha}, \bar{N}_{\alpha+A_{\mu}}) \quad (13)$$

can accommodate the three generations of quarks and leptons.

It is not trivial to accomplish all of these requirements so let me demonstrate by an explicit example.

For the embedding I take $A_{\mu} = (1, 2, 3, 6)$ and for the quiver nodes take the ordering:

$$-C - W - H - C - W^4 - H^4 - \quad (14)$$

with the two ends of (14) identified.

The scalars follow from $a_i = (3, 4, 5)$ and the scalars in Eq.(12)

$$\Sigma_{i=1}^{i=3} \Sigma_{\alpha=1}^{\alpha=12} (3_{\alpha}, \bar{3}_{\alpha \pm a_i}) \quad (15)$$

are sufficient to break to all diagonal subgroups as

$$SU(3)_C \times SU(3)_W \times SU(3)_H \quad (16)$$

The fermions follow from A_{μ} in Eq.(13) as

$$\Sigma_{\mu=1}^{\mu=4} \Sigma_{\alpha=1}^{\alpha=12} (3_{\alpha}, \bar{3}_{\alpha+A_{\mu}}) \quad (17)$$

and the particular dodecahedral quiver in (14) gives rise to exactly *three* chiral generations which transform under (16) as

$$3[(3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3})] \quad (18)$$

I note that anomaly freedom of the underlying superstring dictates that only the combination of states in Eq.(18) can survive. Thus, it is sufficient to examine one of the terms, say $(3, \bar{3}, 1)$. By drawing the quiver diagram indicated by Eq.(14) with the twelve nodes on a “clock-face” and using $A_{\mu} = (1, 2, 3, 6)$ in Eq.(7) I find five $(3, \bar{3}, 1)$ ’s and two $(\bar{3}, 3, 1)$ ’s implying three chiral families as stated in Eq.(18).

After further symmetry breaking at scale M to $SU(3)_C \times SU(2)_L \times U(1)_Y$ the surviving chiral fermions are the quarks and leptons of the SM. The appearance of three families depends on both the identification of modes in (14) and on the embedding of $\Gamma \subset SU(4)$. The embedding must simultaneously give adequate scalars whose VEVs can break the symmetry spontaneously to (16). All of this is achieved successfully by the choices made. The three gauge couplings evolve according to Eqs.(9,10,11) for $M_Z \leq \mu \leq M$. For $\mu \geq M$ the (equal) gauge couplings of $SU(3)^{12}$ do not run if, as conjectured in ^{2,8} there is a conformal fixed point at $\mu = M$.

The basis of the conjecture in ^{2,8} is the proposed duality of Maldacena¹ which shows that in the $N \rightarrow \infty$ limit $N = 4$ supersymmetric $SU(N)$ gauge theory, as well as orbifolded versions with $N = 2, 1$ and 0 ^{20,21} become conformally invariant. It was known long ago ⁹ that the $N = 4$ theory is conformally invariant for all finite $N \geq 2$. This led to the conjecture in ² that the $N = 0$ theories might be conformally invariant, at least in some case(s), for finite N . It should be emphasized that this conjecture cannot be checked purely within a perturbative framework¹⁹. I assume that the local $U(1)$'s which arise in this scenario and which would lead to $U(N)$ gauge groups are non-dynamical, as suggested by Witten²², leaving $SU(N)$'s.

As for experimental tests of such a TeV GUT, the situation at energies below 4 TeV is predicted to be the standard model with a Higgs boson still to be discovered at a mass predicted by radiative corrections ²³ to be below 267 GeV at 99% confidence level.

There are many particles predicted at $\simeq 4$ TeV beyond those of the minimal standard model. They include as spin-0 scalars the states of Eq.(15). and as spin-1/2 fermions the states of Eq.(17). Also predicted are gauge bosons to fill out the gauge groups of (16), and in the same energy region the gauge bosons to fill out all of $SU(3)$ ¹². All these extra particles are necessitated by the conformality constraints of ^{2,8} to lie close to the conformal fixed point.

One important issue is whether this proliferation of states at ~ 4 TeV is compatible with precision electroweak data in hand. This has been studied in the related model of ¹⁸ in a recent article²⁴. Those results are not easily translated to the present model but it is possible that such an analysis including limits on flavor-changing neutral currents could rule out the entire framework.

As alternative to $SU(3)$ ¹² another approach to TeV unification has as its group at ~ 4 TeV $SU(6)$ ³ where one $SU(6)$ breaks diagonally to color while the other two $SU(6)$'s each break to $SU(3)_{k=5}$ where level $k = 5$ characterizes irregular embedding²⁵. The triangular quiver $-C - W - H -$ with ends identified and $A_\mu = (\alpha, \alpha, \alpha, 1)$, $\alpha = \exp(2\pi i/3)$, preserves $N = 1$ supersymmetry. I have chosen to describe the $N = 0$ $SU(3)$ ¹² model in the text mainly because the symmetry breaking to the standard model is more transparent.

The TeV unification fits $\sin^2\theta$ and α_3 , predicts three families, and partially resolves the GUT hierarchy. If such unification holds in Nature there is a very rich level of physics one order of magnitude above presently accessible energy.

Is a hierarchy problem resolved in the present theory? In the non-

gravitational limit $M_{Planck} \rightarrow \infty$ I have, above the weak scale, the new unification scale ~ 4 TeV. Thus, although not totally resolved, the GUT hierarchy is ameliorated.

4. Predictivity

The calculations have been done in the one-loop approximation to the renormalization group equations and threshold effects have been ignored. These corrections are not expected to be large since the couplings are weak in the entire energy range considered. There are possible further corrections such as non-perturbative effects, and the effects of large extra dimensions, if any.

In one sense the robustness of this TeV-scale unification is almost self-evident, in that it follows from the weakness of the coupling constants in the evolution from M_Z to M_U . That is, in order to define the theory at M_U , one must combine the effects of threshold corrections (due to $O(\alpha(M_U))$ mass splittings) and potential corrections from redefinitions of the coupling constants and the unification scale. We can then *impose* the coupling constant relations at M_U as renormalization conditions and this is valid to the extent that higher order corrections do not destabilize the vacuum state.

We shall approach the comparison with data in two different but almost equivalent ways. The first is "bottom-up" where we use as input that the values of $\alpha_3(\mu)/\alpha_2(\mu)$ and $\sin^2 \theta(\mu)$ are expected to be 5/2 and 1/4 respectively at $\mu = M_U$.

Using the experimental ranges allowed for $\sin^2 \theta(M_Z) = 0.23113 \pm 0.00015$, $\alpha_3(M_Z) = 0.1172 \pm 0.0020$ and $\alpha_{em}^{-1}(M_Z) = 127.934 \pm 0.027$ ²³ we have calculated²⁸ the values of $\sin^2 \theta(M_U)$ and $\alpha_3(M_U)/\alpha_2(M_U)$ for a range of M_U between 1.5 TeV and 8 TeV. Allowing a maximum discrepancy of $\pm 1\%$ in $\sin^2 \theta(M_U)$ and $\pm 4\%$ in $\alpha_3(M_U)/\alpha_2(M_U)$ as reasonable estimates of corrections, we deduce that the unification scale M_U can lie anywhere between 2.5 TeV and 5 TeV. Thus the theory is robust in the sense that there is no singular limit involved in choosing a particular value of M_U .

Another test of predictivity of the same model is to fix the unification values at M_U of $\sin^2 \theta(M_U) = 1/4$ and $\alpha_3(M_U)/\alpha_2(M_U) = 5/2$. We then compute the resultant predictions at the scale $\mu = M_Z$.

The results are shown for $\sin^2 \theta(M_Z)$ in Fig. 1 with the allowed range²³ $\alpha_3(M_Z) = 0.1172 \pm 0.0020$. The precise data on $\sin^2(M_Z)$ are indicated in Fig. 1 and the conclusion is that the model makes correct predictions for $\sin^2 \theta(M_Z)$. Similarly, in Fig 2, there is a plot of the prediction for

$\alpha_3(M_Z)$ versus M_U with $\sin^2 \theta(M_Z)$ held with the allowed empirical range. The two quantities plotted in Figs 1 and 2 are consistent for similar ranges of M_U . Both $\sin^2 \theta(M_Z)$ and $\alpha_3(M_Z)$ are within the empirical limits if $M_U = 3.8 \pm 0.4$ TeV.

The model has many additional gauge bosons at the unification scale, including neutral Z' 's, which could mediate flavor-changing processes on which there are strong empirical upper limits.

A detailed analysis will require specific identification of the light families and quark flavors with the chiral fermions appearing in the quiver diagram for the model. We can make only the general observation that the lower bound on a Z' which couples like the standard Z boson is quoted as $M(Z') < 1.5$ TeV²³ which is safely below the M_U values considered here and which we identify with the mass of the new gauge bosons.

This is encouraging to believe that flavor-changing processes are under control in the model but this issue will require more careful analysis when a specific identification of the quark states is attempted.

Since there are many new states predicted at the unification scale ~ 4 TeV, there is a danger of being ruled out by precision low energy data. This issue is conveniently studied in terms of the parameters S and T introduced in²⁶ and designed to measure departure from the predictions of the standard model.

Concerning T , if the new $SU(2)$ doublets are mass-degenerate and hence do not violate a custodial $SU(2)$ symmetry they contribute nothing to T . This therefore provides a constraint on the spectrum of new states.

According to²³, a multiplet of degenerate heavy chiral fermions gives a contribution to S :

$$S = C \sum_i (t_{3L}(i) - t_{3R}(i))^2 / 3\pi \quad (19)$$

where $t_{3L,R}$ is the third component of weak isospin of the left- and right-handed component of fermion i and C is the number of colors. In the present model, the additional fermions are non-chiral and fall into vector-like multiplets and so do not contribute to Eq.(19).

Provided that the extra isospin multiplets at the unification scale M_U are sufficiently mass-degenerate, therefore, there is no conflict with precision data at low energy.

5. Discussion

The plots we have presented clarify the accuracy of the predictions of this TeV unification scheme for the precision values accurately measured at the Z-pole. The predictivity is as accurate for $\sin^2 \theta$ as it is for supersymmetric GUT models^{7,14,27,29}. There is, in addition, an accurate prediction for α_3 which is used merely as input in SusyGUT models.

At the same time, the accurate predictions are seen to be robust under varying the unification scale around $\sim 4TeV$ from about 2.5 TeV to 5 TeV.

In conclusion, since this model ameliorates the GUT hierarchy problem and naturally accommodates three families, it provides a viable alternative to the widely-studied GUT models which unify by logarithmic evolution of couplings up to much higher GUT scales.

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Figure Captions

Fig. 1.

Plot of $\sin^2 \theta(M_Z)$ versus M_U in TeV, assuming $\sin^2 \theta(M_U) = 1/4$ and $\alpha_3/\alpha_2(M_U) = 5/2$.

Fig.2.

Plot of $\alpha_3(M_Z)$ versus M_U in TeV, assuming $\sin^2 \theta(M_U) = 1/4$ and $\alpha_3/\alpha_2(M_U) = 5/2$.

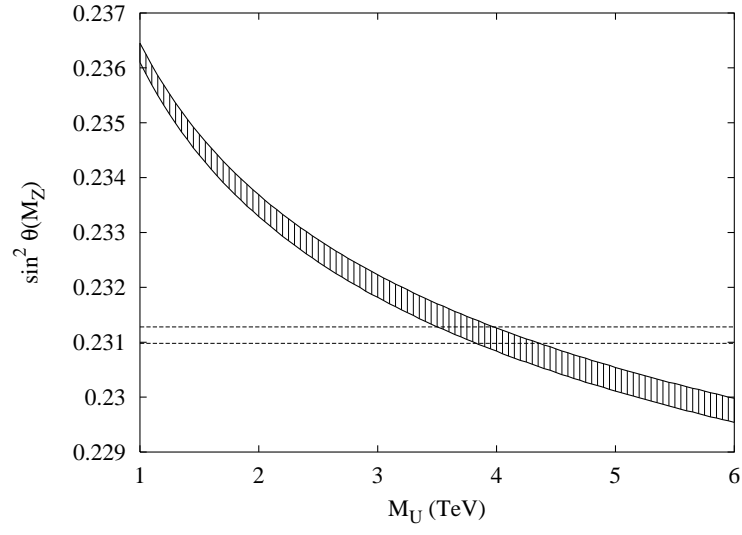


Figure 1.

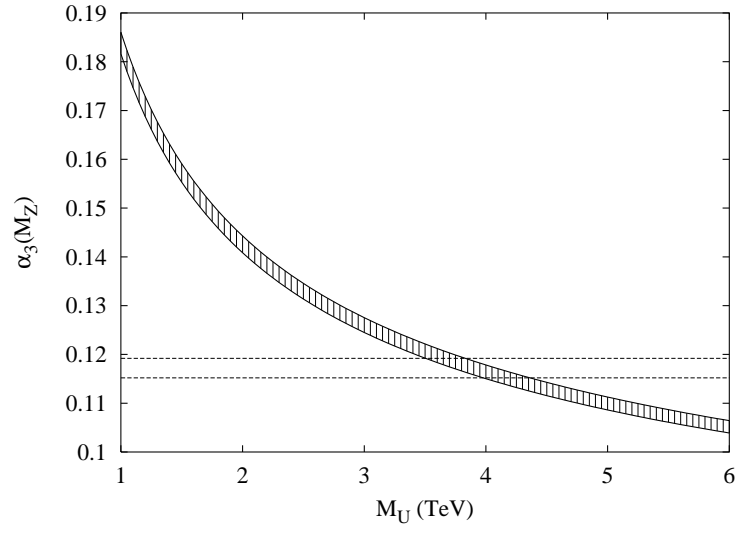


Figure 2.

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